

NAG Toolbox for MATLAB

f08ka

1 Purpose

f08ka computes the minimum norm solution to a real linear least-squares problem

$$\min_x \|b - Ax\|_2.$$

2 Syntax

```
[a, b, s, rank, info] = f08ka(a, b, rcond, 'm', m, 'n', n, 'nrhs_p',
nrhs_p)
```

3 Description

f08ka uses the singular value decomposition (SVD) of A , where A is an m by n matrix which may be rank-deficient.

Several right-hand side vectors b and solution vectors x can be handled in a single call; they are stored as the columns of the m by r right-hand side matrix B and the n by r solution matrix X .

The effective rank of A is determined by treating as zero those singular values which are less than **rcond** times the largest singular value.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: <http://www.netlib.org/lapack/lug>

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **a(lda,*)** – double array

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The m by n matrix A .

2: **b(lb,*)** – double array

The first dimension of the array **b** must be at least $\max(1, \max(\mathbf{m}, \mathbf{n}))$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs_p})$

The m by r right-hand side matrix B .

3: **rcond** – double scalar

Used to determine the effective rank of A . Singular values $\mathbf{s}(i) \leq \mathbf{rcond} \times \mathbf{s}(1)$ are treated as zero. If **rcond** < 0, *machine precision* is used instead.

5.2 Optional Input Parameters

1: **m – int32 scalar**

Default: The first dimension of the array **a**.

m , the number of rows of the matrix A .

Constraint: $m \geq 0$.

2: **n – int32 scalar**

Default: The second dimension of the array **a**.

n , the number of columns of the matrix A .

Constraint: $n \geq 0$.

3: **nrhs_p – int32 scalar**

Default: The second dimension of the array **b**.

r , the number of right-hand sides, i.e., the number of columns of the matrices B and X .

Constraint: **nrhs_p** ≥ 0 .

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldb, work, lwork

5.4 Output Parameters

1: **a(lda,*) – double array**

The first dimension of the array **a** must be at least $\max(1, m)$

The second dimension of the array must be at least $\max(1, n)$

The first $\min(m, n)$ rows of A are overwritten with its right singular vectors, stored row-wise.

2: **b(ldb,*) – double array**

The first dimension of the array **b** must be at least $\max(1, \max(m, n))$

The second dimension of the array must be at least $\max(1, \text{nrhs_p})$

b contains the n by r solution matrix X . If $m \geq n$ and **rank** = n , the residual sum of squares for the solution in the i th column is given by the sum of squares of elements $n + 1, \dots, m$ in that column.

3: **s(*) – double array**

Note: the dimension of the array **s** must be at least $\max(1, \min(m, n))$.

The singular values of A in decreasing order.

4: **rank – int32 scalar**

The effective rank of A , i.e., the number of singular values which are greater than **rcond** $\times$ **s**(1).

5: **info – int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **m**, 2: **n**, 3: **nrhs_p**, 4: **a**, 5: **lda**, 6: **b**, 7: **ldb**, 8: **s**, 9: **rcond**, 10: **rank**, 11: **work**, 12: **lwork**, 13: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

The algorithm for computing the SVD failed to converge; if **info** = i , i off-diagonal elements of an intermediate bidiagonal form did not converge to zero.

7 Accuracy

See Section 4.5 of Anderson *et al.* 1999 for details.

8 Further Comments

The complex analogue of this function is f08kn.

9 Example

```
a = [-0.09, 0.14, -0.46, 0.68, 1.29;
      -1.56, 0.2, 0.29, 1.09, 0.51;
      -1.48, -0.43, 0.89, -0.71, -0.96;
      -1.09, 0.84, 0.77, 2.11, -1.27;
      0.08, 0.55, -1.13, 0.14, 1.74;
      -1.59, -0.72, 1.06, 1.24, 0.34];
b = [7.4;
      4.2;
      -8.3000000000000001;
      1.8;
      8.6;
      2.1];
rcond = 0.01;
[aOut, bOut, s, rank, info] = f08ka(a, b, rcond)
```

```
aOut =
    0.6554    0.0104   -0.4376   -0.5300    0.3130
    0.1391   -0.1857    0.2942   -0.5255   -0.7638
    0.5134    0.5066   -0.1540    0.5115   -0.4409
    0.5064   -0.6588    0.3827    0.3792    0.1389
   -0.1765   -0.5241   -0.7428    0.1934   -0.3239
    0.5335   -0.1595   -0.2363   -0.3490   -0.6641

bOut =
    0.6344
    0.9699
   -1.4403
    3.3678
    3.3992
   -0.0035

s =
    3.9997
    2.9962
    2.0001
    0.9988
```

	0.0025
rank =	
info =	4
	0
